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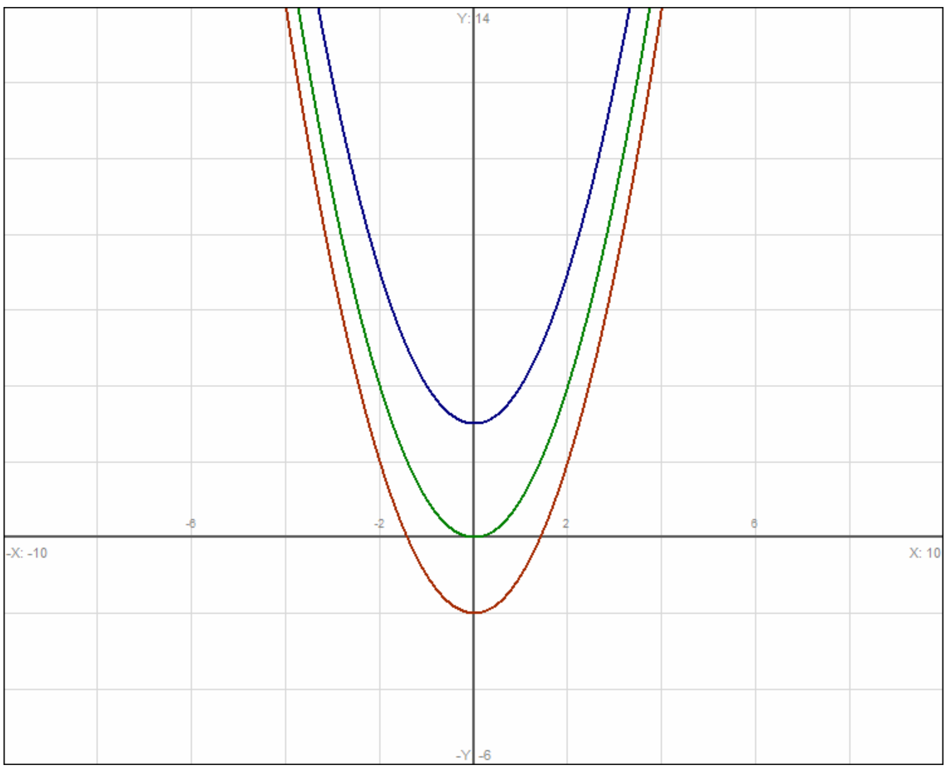
Math Methods 11

Portfolio Assignment 1

INVESTIGATING THE QUADRATIC FUNCTION

1. Sketch the graphs of:

   a) $y = x^2$
   b) $y = x^2 + 3$
   c) $y = x^2 - 2$



Graph 1 x [-10,10]; y [-6,14]

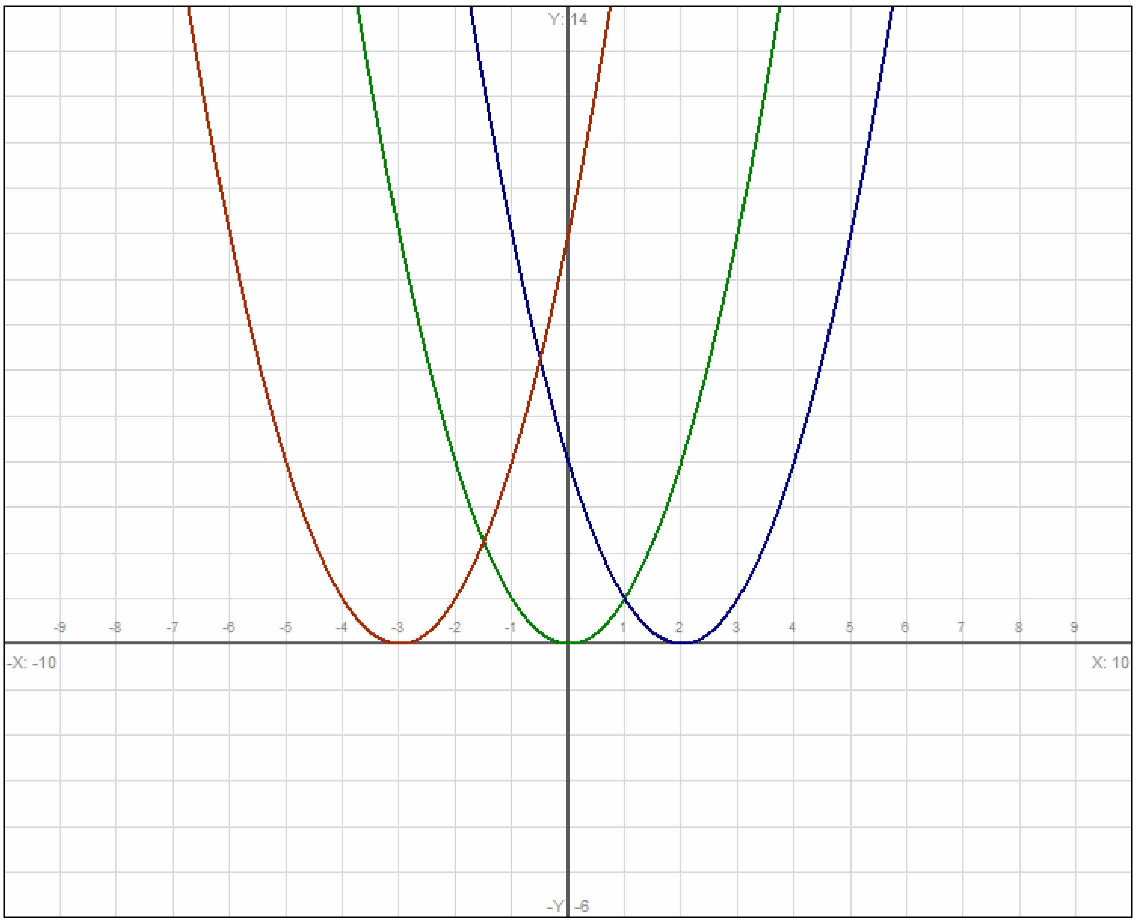
All three graphs have been graphed in the same screen for comparison. From the above graphs, we notice that the graphs of b) $y = x^2 + 3$ and c) $y = x^2 - 2$ have been shifted vertically, either up or down by *k* units, where *k* is the number that follows x^2.

By looking at the graphs we can generalise the following:

The graph of $f(x) = x^2 + k$ is the graph of $f(x) = x^2$, vertically translated *k* units. If *k* is positive, then the shift is upwards. Conversely, if *k* is negative, the shift is downwards.

2. Consider the graphs of:

 a) $y = x^2$
 b) $y = (x-2)^2$
 c) $y = (x+3)^2$



Graph 2 *x* [-10,10]; y [-6,14]

As in the previous case, all three graphs have been graphed in the same screen for comparison. From the above graphs, we notice that the graphs of b) $y = (x-2)^2$ and c) $y = (x+3)^2$ have been shifted horizontally, either left or right by *h* units, where *h* is the number that follows after $x -$.

By looking at the graphs we can generalise the following:

The graph of $f(x)=(x-h)^2$ is the graph of $f(x)=x^2$, horizontally translated *h* units. If *h* is positive, the shift is to the right. Conversely, if *h* is negative, the shift is to the left.

Note: In part b), the value of *h* is 2, and not -2, since our standard form is $f(x)=(x-h)^2$. In part c), the equation $y=(x+3)^2$ can be converted to $y=(x-(-3))^2$, to suit our format. Here, $h=-3$.

3. Where would you expect the vertex on the graph of $y=(x-4)^2+5$ to be? Explain why.

   The graph of $y=(x-4)^2+5$ is the graph of $y=x^2$, vertically translated 5 units upward, and horizontally translated 4 units to the right. Since the vertex of the graph $y=x^2$ is at (0,0), the vertex of the graph $y=(x-4)^2+5$ should be at (4,5).

   Alternatively, since we know that the vertex of a parabolic function of the form $f(x)=(x-h)^2+k$ lies at (*h*,*k*), we can expect the vertex of the graph $y=(x-4)^2+5$ to be at (4,5) as *h*=4 and *k*=5.

4. a) Express $x^2-10x+25$ in the form $(x-h)^2$.

   $x^2-10x+25$ is noticed to be of the form $a^2-2ab+b^2$.
   As $a^2-2ab+b^2=(a-b)^2$, $\underline{x^2-10x+25=(x-5)^2}$

   b) Express $x^2-10x+32$ in the form $(x-h)^2+g$

   To convert from one form to the other, we use the method of completion of squares.

   $f(x)=x^2-10x+32$

   $f(x)-32=x^2-10x$ Subtracting 32 from both sides.

   $f(x)-32+\left(\frac{-10}{2}\right)^2=x^2-10x+\left(\frac{-10}{2}\right)^2$ Adding 25 to both sides.

   $f(x)-7=(x-5)^2$ as $a^2-2ab+b^2=(a-b)^2$

   $f(x)=(x-5)^2+7$ Adding 7 to both sides.

   c) Repeat this procedure with some examples of your own.

   i) Converting x^2-4x+3 to the form $(x-h)^2+g$

$$f(x) = x^2 - 4x + 3$$
$$f(x) - 3 = x^2 - 4x$$
$$f(x) - 3 + \left(\frac{-4}{2}\right)^2 = x^2 - 4x + \left(\frac{-4}{2}\right)^2$$
$$f(x) + 1 = (x-2)^2$$
$$f(x) = (x-2)^2 - 1$$

ii) Converting $x^2 + 12x + 16$ to the form $(x-h)^2 + g$

$$f(x) = x^2 + 12x + 16$$
$$f(x) - 16 = x^2 + 12x$$
$$f(x) - 16 + \left(\frac{12}{2}\right)^2 = x^2 + 12x + \left(\frac{12}{2}\right)^2$$
$$f(x) + 20 = (x+6)^2$$
$$f(x) = (x+6)^2 - 20$$

d) Describe a method of writing the quadratic expression $x^2 + bx + c$ in the form $(x-h)^2 + g$.

1. Let the expression equal to *y*.
2. Isolate the *x* terms.
3. Add $\left(\frac{b}{2}\right)^2$ to both sides.
4. Factor the perfect square trinomial thus obtained.
5. Isolate the *y*.
6. The equation is now in the form $(x-h)^2 + g$.

5. Describe the shape and position of the graph $y = (x-h)^2 + g$. Provide an explanation for this.

The shape of the graph of $y = (x-h)^2 + g$ will be parabolic, since it is a quadratic equation. We know that the equation is quadratic, due to the presence of a squared expression. The shape will be the same as the graph of $y = x^2$, as there isn't any horizontal or vertical compression or expansion.

The graph of $y = (x-h)^2 + g$ is the graph of $y = x^2$, vertically translated *g* units, and horizontally translated *h* units.

Hence the graph of $y = (x-h)^2 + g$ will be exactly similar to the graph of $y = x^2$ except that it will be moved *h* units right or left (right if h>0 and left if h<0), and *g* units up or down (up if g>0 and down if g<0).

6. Do your findings apply to the graphs of other type of functions? Can you generalise?

   Yes, the above findings apply to any type of function of the form $y = f(x)$.

   Some of the commonly used functions are:

   1. Linear Function: $f(x) = x$
   2. Quadratic Function: $f(x) = x^2$
   3. Absolute Value Function: $f(x) = |x|$
   4. Cubic Function: $f(x) = x^3$
   5. Rational Function: $f(x) = \frac{1}{x}$
   6. Radical Function: $f(x) = \sqrt{x}$
   7. Exponential Function: $f(x) = 2^x$

   We can generalise translations with the following rule:

   The graph of the any function of the form $y = f(x-h)+g$, is the graph of the function $y = f(x)$, translated *h* units horizontally (right if h>0 and left if h<0), and translated *g* units vertically (up if g>0 and down if g<0).

Note: All graphs contained within this assignment are generated with:

GraphCalc
Version 4.0 Alpha
http://www.graphcalc.com/