A Sequential Linear Programming Algorithm for Portfolio Optimization at GE

MSCI 331 Operations Research 1

Richard Hui, Rajesh Swaminathan, Sarah Vandaiyar

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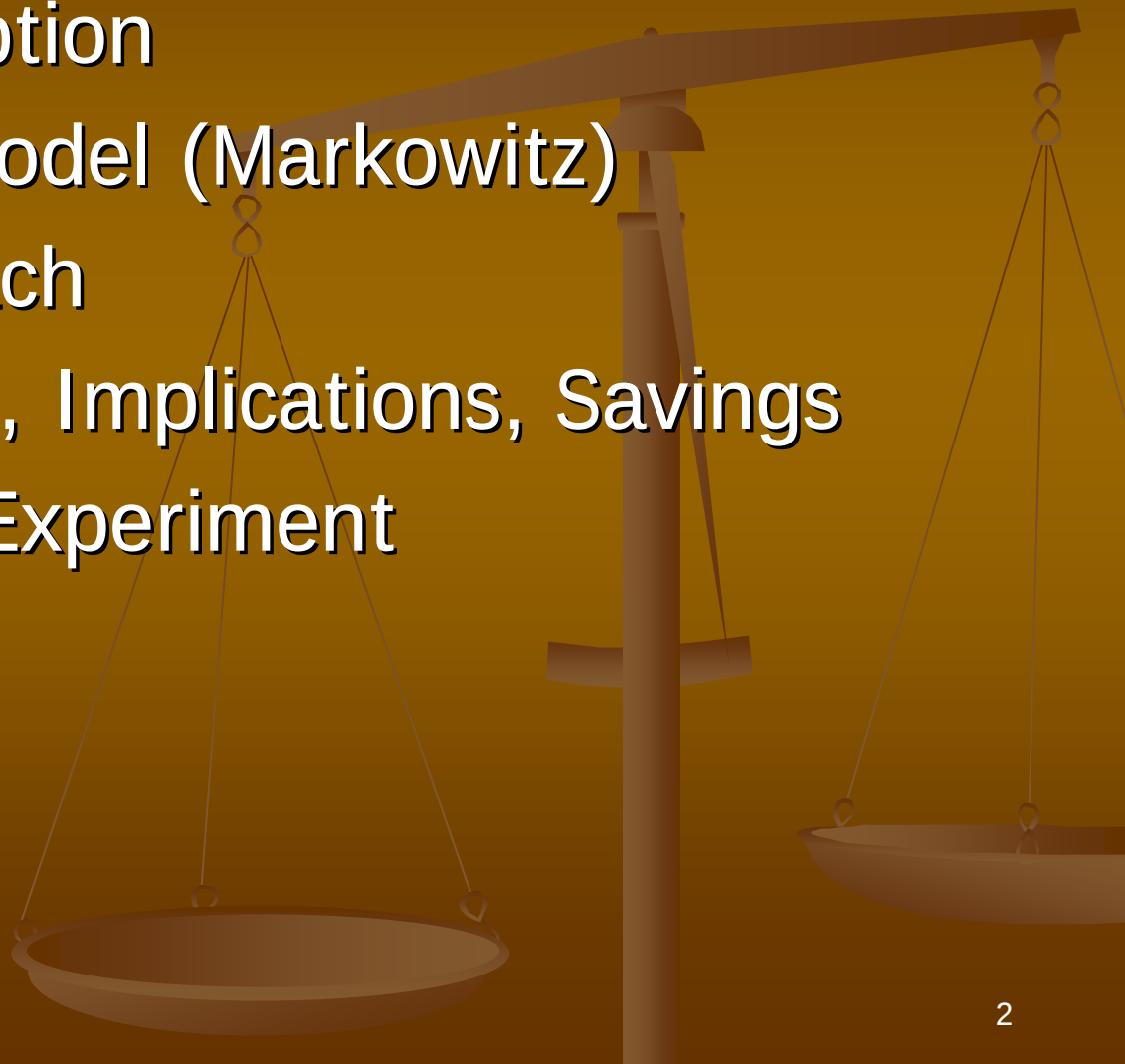
Discussion Plan

- Problem Description
- Mathematical Model (Markowitz)
- Solution Approach
- Implementation, Implications, Savings
- Computational Experiment
- Q & A



Overview of Portfolio Optimization

- Definitions:
  - Portfolio
  - Risk
  - Return
  - Correlation
  - Asset weights
- GE Asset Management manages investment portfolios on behalf of clients

Problem at GE

- Optimization of non-linear problems was beyond computational limits of solvers
- Concerns posed by simple LP approximations
- Previous algorithms, because of time constraints, could be run only after market variables changed, not before

Assumptions

- Investors are risk averse; they prefer less risk to more for the same level of expected return
- Expected returns, variances and covariances of all assets are known
- Ignore skew and kurtosis of distribution
- No transaction costs or taxes

Mathematical Model

- Decision Variables

  Asset weights w_1, w_2, and w_3

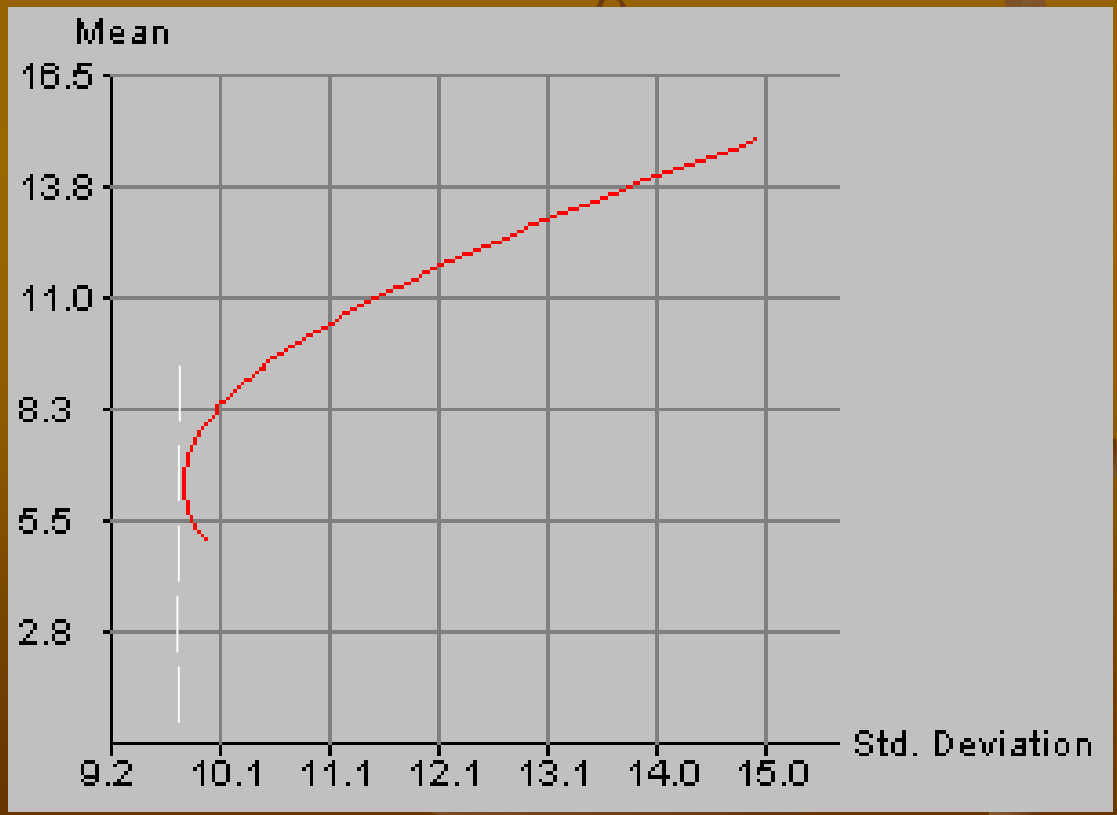
- Objective Function

$$\text{MIN } \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(R_i R_j)$$

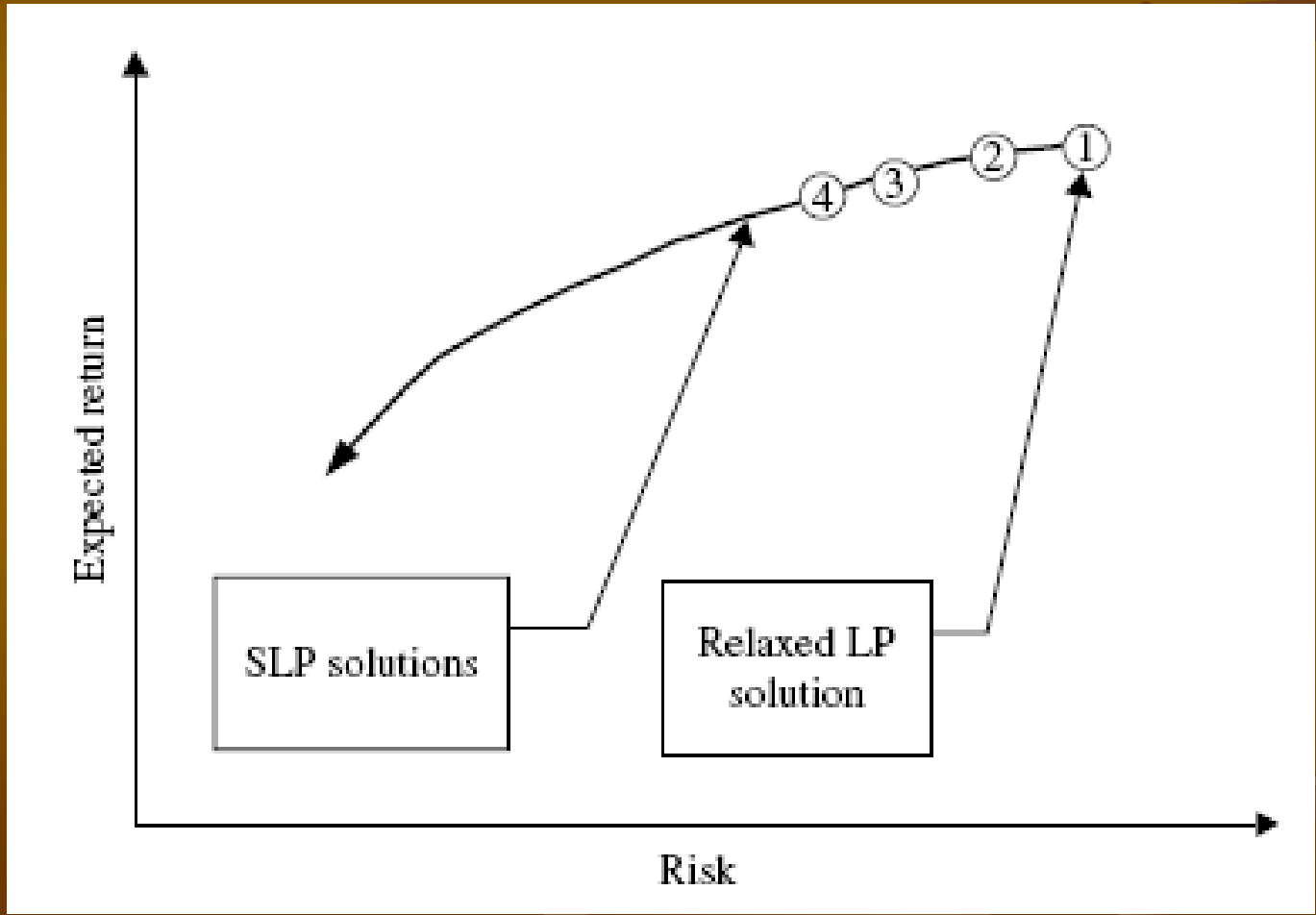
- Constraints
  - Weight constraints $\sum_{j=1}^{n} w_j = 1$
  - Fixed return $E(R_p) = \sum_{j=1}^{n} w_j E(R_j) = z$
  - Non-negativity $w_j \geq 0,\ j = 1, 2, 3, \ldots$

Markowitz Model

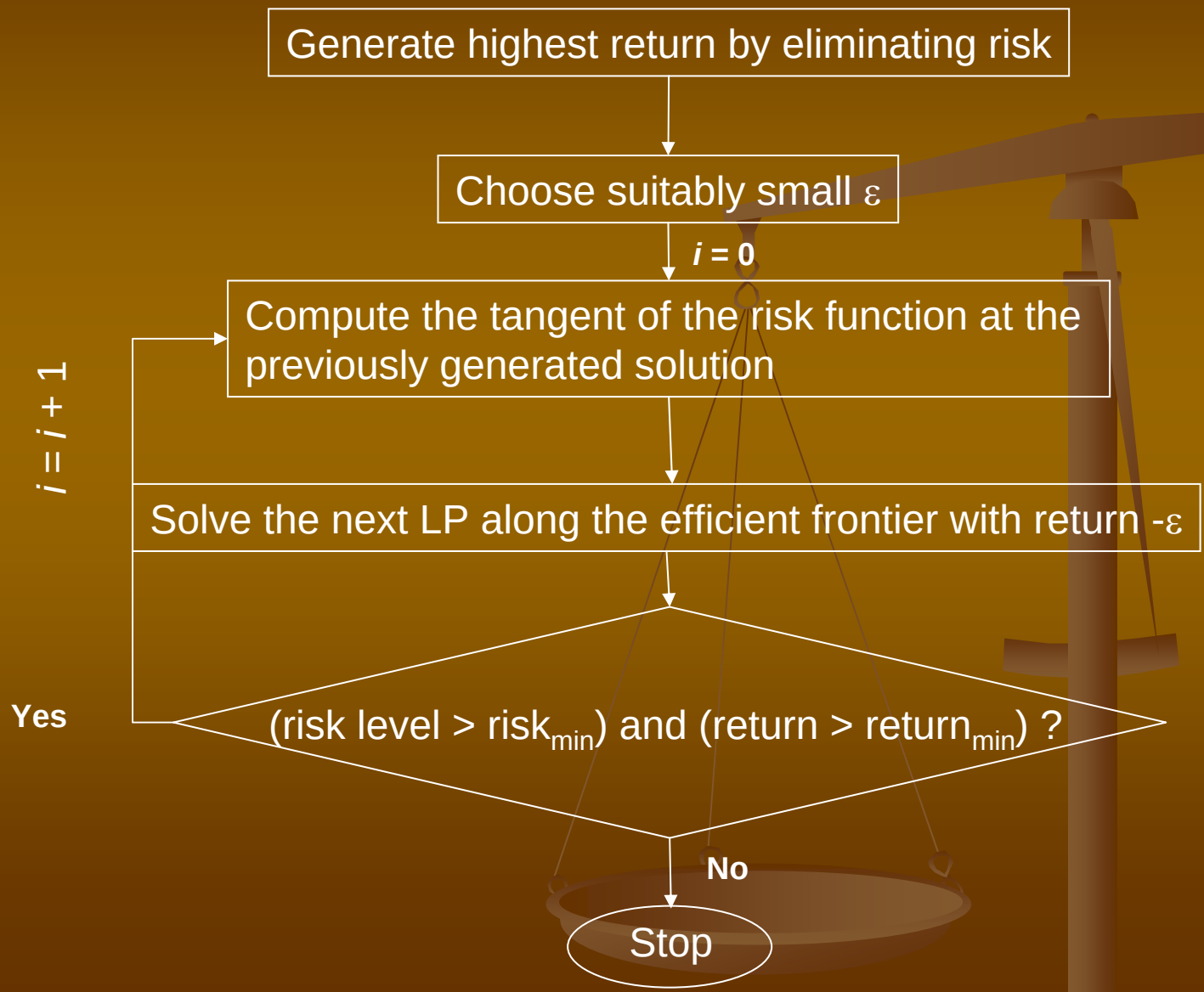
- Objective: Generate the *minimum variance frontier*
- Find the lowest risk for a given level of return



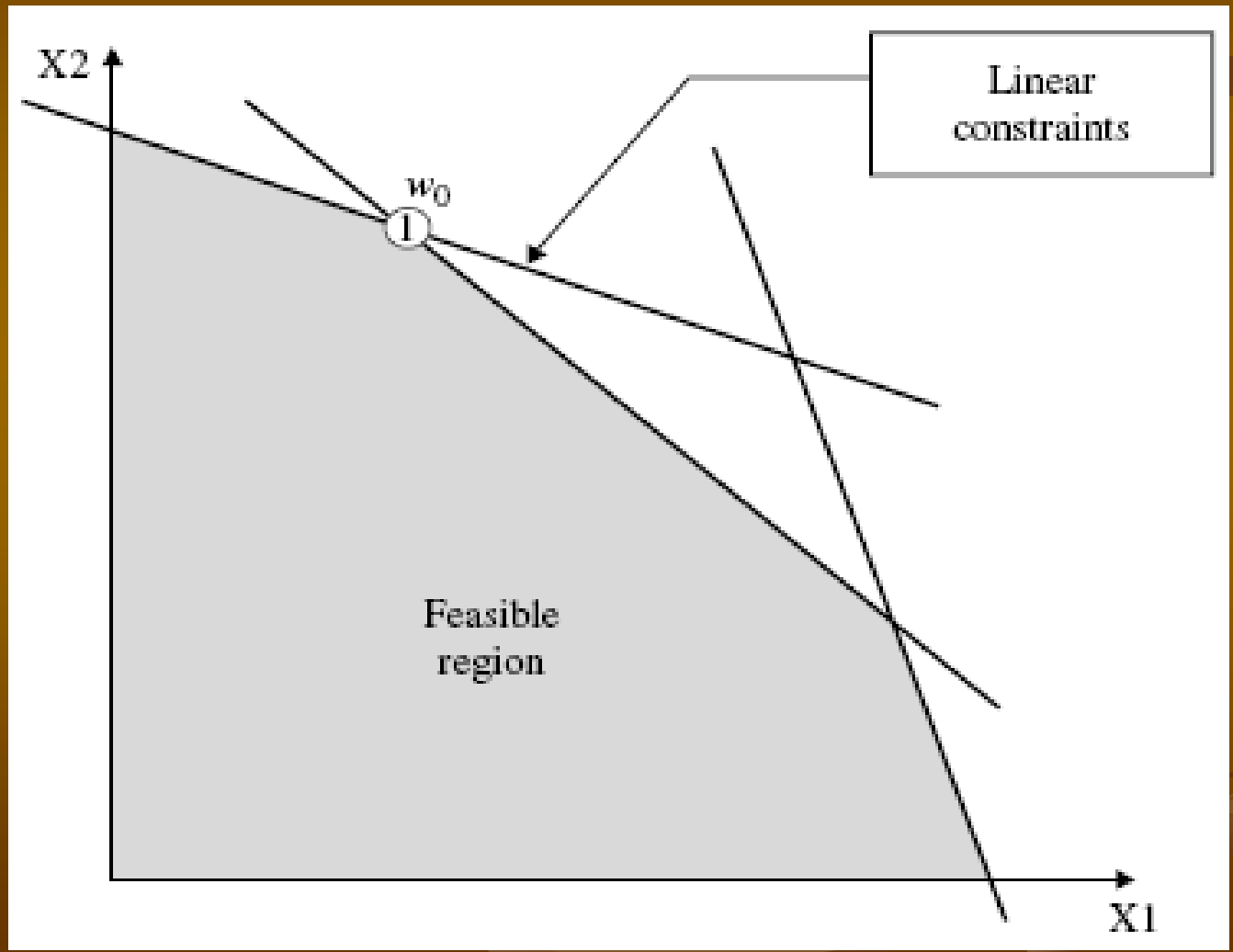
Sequential Linear Programming (SLP) Overview



SLP Algorithm



SLP Example for 2 Assets



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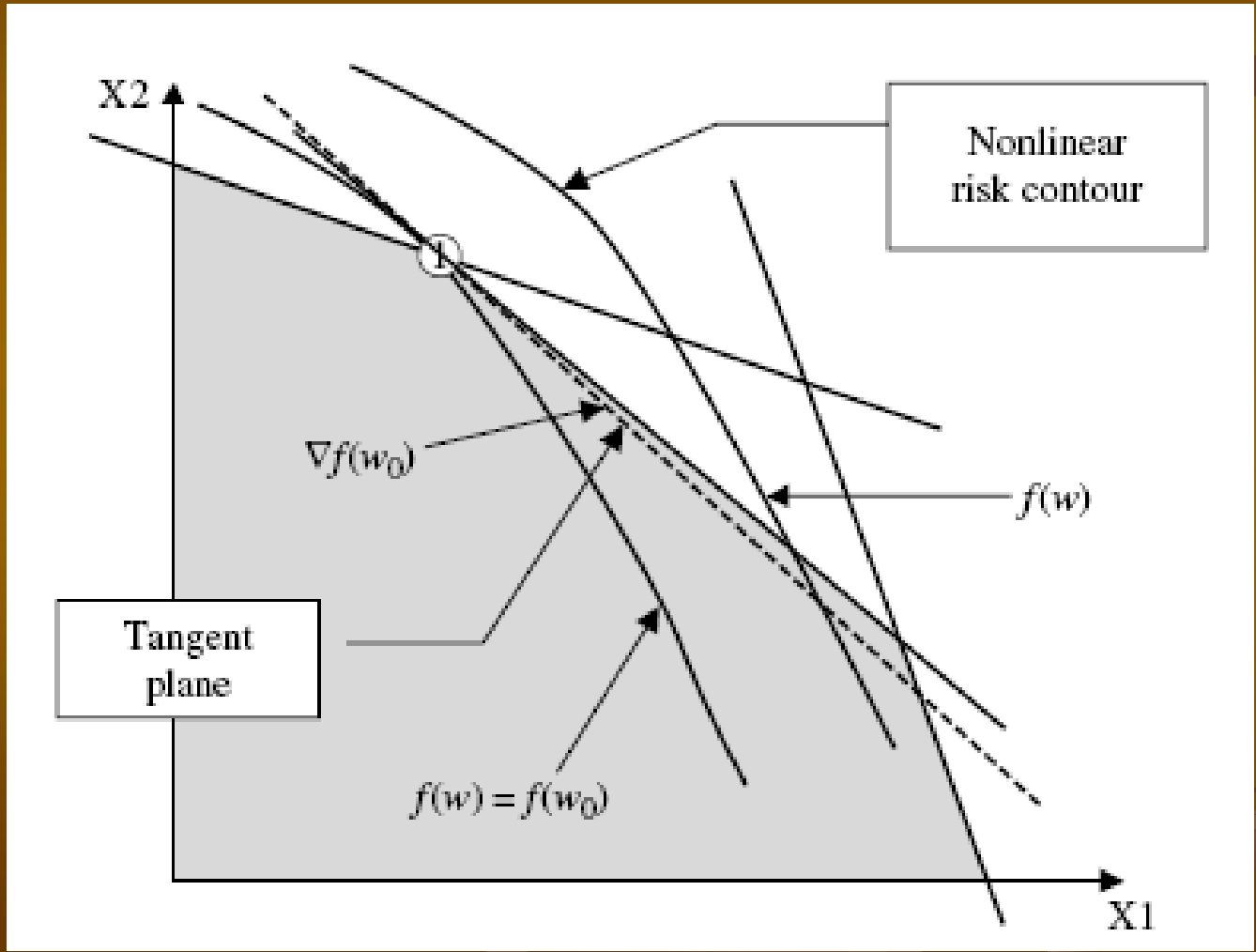
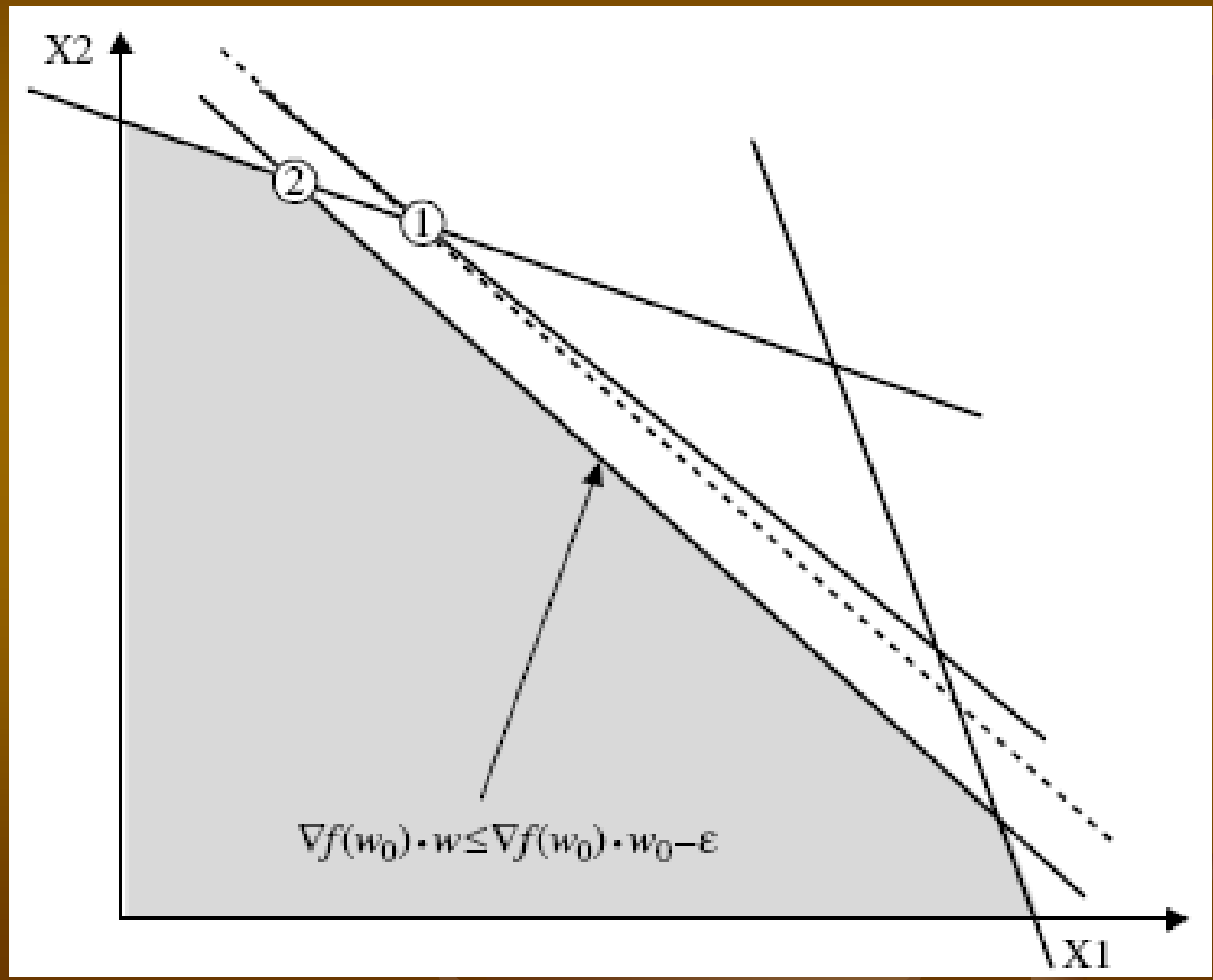


SLP Example for 2 Assets



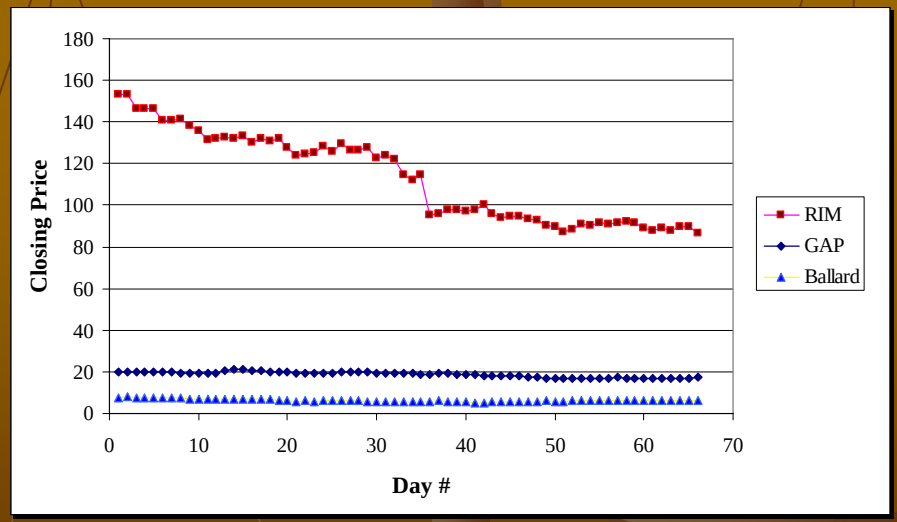
Implementation and Savings

- Implementation
  - Algorithm coded in MATLAB
  - Web-based Java interface using JMatLink
  - Deployed on GE's Intranet
- Benefits
  - Polynomial-time versus exponential-time
  - Managers can run algorithm multiple times
  - Optimized over 30 portfolios valued at $30 billion
  - Expected benefits total $75 million over five years
- Limitations
  - Uses historical data to predict future trends
  - Model works only for convex risk functions

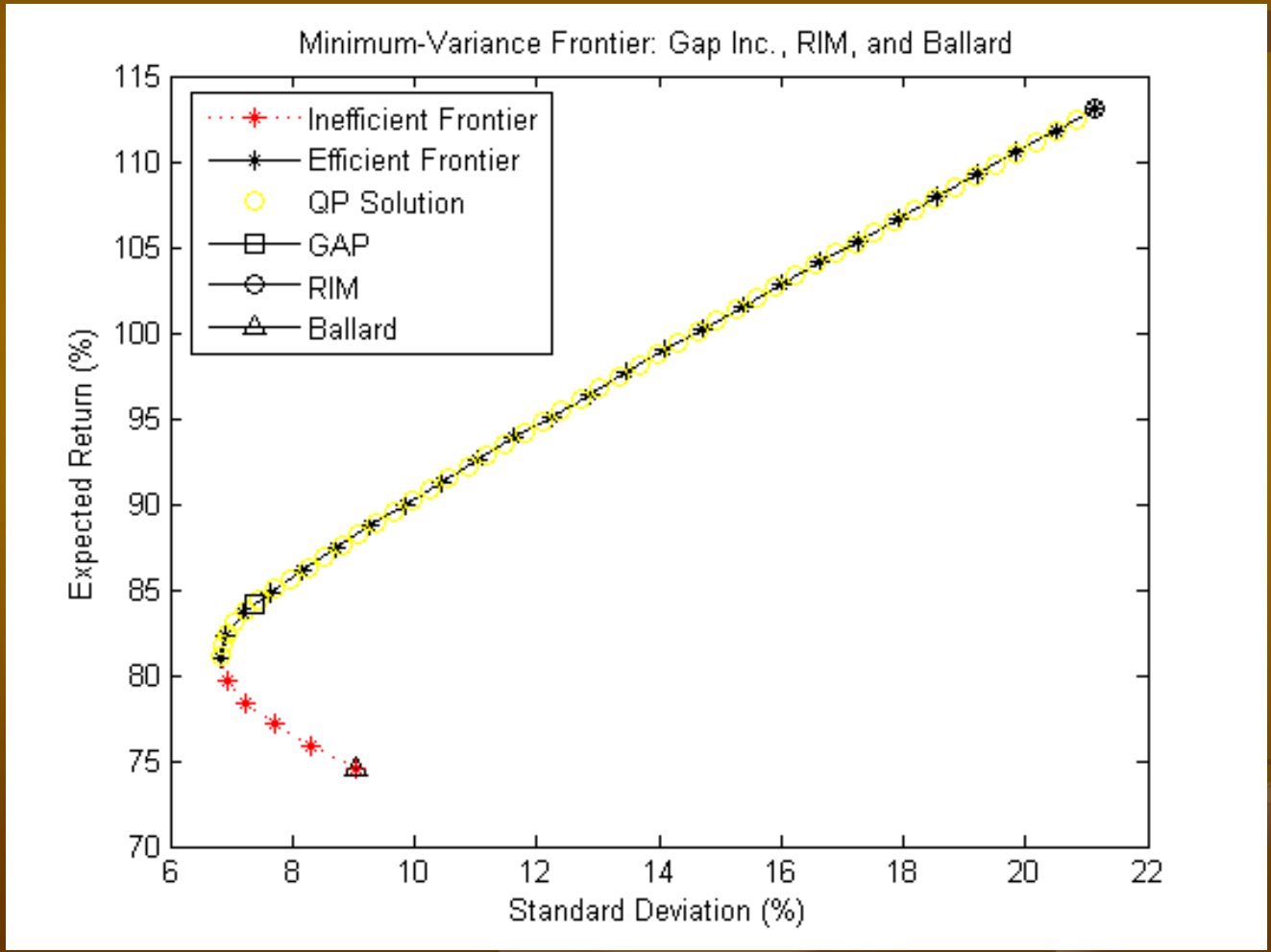
Computational Experiment

- Choice of Stocks: RIM, Gap, Ballard
- Coded in MATLAB using `linprog`
- Minimum Variance Frontier
- Linear Risk Approximation (Taylor)
- Advantages to Diversification

	GAP	RIM	Ballard
Mean Daily Returns	18.70	113.10	6.21
Standard Sample Deviation	1.341	21.151	0.645
Correlations			
GAP	1.000		
RIM	0.883	1.000	
Ballard	0.455	0.683	1.000



Closeness of Approximation LP vs. QP



References

- Chalermkraivuth et al. “Sequential-Linear-Programming Algorithm to Optimize Portfolios.” *Interfaces* 35(5), pp. 370-380. Informs, 2005.
- DeFusco et al. “Quantitative Methods For Investment Analysis.” 2nd Ed. *CFA Institute,* 2005.
- Markowitz, H. “Portfolio Finance.” *J. Finance* 7(1) pp. 77-91. 1952.

Questions?