Physics 12 IB **Waves and Beats**

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6.7.2 Derivation of the Beat Frequency Formula

If we consider just one point on a one-dimensional sinusoidal wave travelling in the x-direction then we can represent the oscillation at that point by a sine function. Let the wave have an amplitude A and frequency f. The displacement y at a point x will vary with time as

$$y = A\sin(2\pi ft)$$

Suppose we now have two waves of the same amplitude travelling in the same direction, one of frequency f_1 and the other of frequency f_2. The displacement at a point x will now be the sum of the individual displacements of the waves (principle of linear superposition). That is

$$Y = A\sin(2\pi f_1 t) + A\sin(2\pi f_2 t)$$

Using the rule for adding sine functions we have

$$Y = 2A\cos(2\frac{\pi}{2}(f_1 - f_2)t)\sin(2\frac{\pi}{2}(f_1 + f_2)t) \tag{1}$$

What this effectively means is that we have a new wave frequency $\frac{1}{2}(f_1 + f_2)$ and with an amplitude which varies periodically with a frequency $f_1 - f_2$. So for example if we sound two tuning forks together, one of frequency 440 Hz and the other of frequency 442 Hz, and both with the same intensity, we will hear a sound of twice the intensity of one of the forks but of frequency 401 Hz. The intensity reaches a maximum intensity twice every second and zero intensity twice every second. That is, the *beat frequency* is 2 Hz.
